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LETTER TO THE EDITOR

A Z_2-gauge symmetry in CPT and the Weinberg–Salam model

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Abstract. By means of an approach to the generalized gauge theory on discrete groups, we present a model of $G_L \times G_R \times Z_2$ gauge invariance where the Higgs field is a gauge field with respect to the Z_2-gauge symmetry, a subgroup of the CPT symmetry. As an example, we reformulate the Weinberg–Salam model.

Very recently, we have generalized [1] the ordinary Yang–Mills gauge theory in order to take not only Lie groups but also discrete groups as gauge groups [2] and completed an approach to this generalized gauge theory coupled to the fermions in the spirit of non-commutative geometry approach [3–7]. We have shown that Higgs fields are such gauge fields with respect to discrete gauge symmetry over 4-dimensional spacetime and the Yukawa couplings between Higgs and fermions may automatically be introduced via covariant derivatives with respect to discrete gauge potentials. We have also presented a toy model with $U_L(1) \times U_R(1) \times Z_2$ gauge symmetry. In this toy model, Higgs appears as a Z_2-gauge field over spacetime M^4 and the assignment of the fields with respect to the elements of the Z_2-guage symmetry for the fermions is related to their chirality and for the gauge bosons and Higgs to the way of their coupling with the fermions. However, since the Weinberg–Salam model and the standard model for the electroweak–strong interaction are not left–right symmetrical, the assignment of the fields in [1] cannot be directly applied to the realistic model building.

In this letter, in order to get rid of this asymmetry problem we take the Z_2-gauge symmetry to be a subgroup of the CPT symmetry, i.e. we take the elements of Z_2, $\{e, r\}$, to be $\{e, r = (CPT)^2\}$, and show that the Higgs sector in the Weinberg–Salam model may be dealt with as the gauge field with respect to this symmetry. In what follows, we first construct a general model with such a gauge symmetry together with a $G_L \times G_R$ gauge symmetry. We assign fermions, gauge bosons and Higgs into two sectors corresponding to two elements of the group Z_2. For the fermions, this assignment links directly to their transformation property under $(\mathrm{CPT})^2$, and for the gauge bosons and Higgs not only to their transformation property under $(\mathrm{CPT})^2$ but also to the method of their coupling to the fermions. As for the Lagrangian, we may obtain the conventional one by consideration of the gauge invariance. On the other hand, however, we may also take into account the Z_2 symmetry to get the Lagrangian for each element of Z_2, first by taking the Haar integral over Z_2 to get the entire Lagrangian. As was mentioned in [1], this may lead to certain constraints among the coupling constants and mass parameters at the tree level under certain

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assumptions. In order to apply to the Weinberg–Salam model, what we need to do is just to take the field content in the general model to be that of the Weinberg–Salam model and release some of the constraints but not the one for the Higgs mass. Finally, we end with some discussions and remarks.

Let us first construct a model of the $G_L \times G_R \times Z_2$-gauge symmetry with $Z_2 \subset$ CPT group with leptons $\psi(x,h), h \in Z_2$, Yang–Mills gauge potentials $A_\mu(x,h)$ and Higgs $\Phi(x,h)$. We assign them into two sectors according to two elements of the group Z_2 as follows:

$$\begin{aligned} \psi(x,e) &= -\psi(x,r) = \begin{pmatrix} L \\ R \end{pmatrix} \\ A_\mu(x,e) &= A_\mu(x,r) = \begin{pmatrix} L_\mu & 0 \\ 0 & R_\mu \end{pmatrix} \\ \Phi(x,e) &= \Phi(x,r) = \begin{pmatrix} \mu/\lambda & -\phi \\ -\phi^\dagger & \mu/\lambda \end{pmatrix} \end{aligned} \tag{1}$$

where $L(R)$ is the left (right) handed fermion, $L_\mu(R_\mu)$ the gauge potential valued on the Lie algebra of the gauge group $G_L(G_R)$ and coupled to the fermion $L(R)$, μ and λ two constants. Note that the minus sign in $-\psi(x,r), r = (CPT)^2$, is due to the transformation property of the fermion under $(\mathrm{CPT})^2$. From the assignment, it is easy to see that the field contents of the model is of Z_2 symmetry and the Higgs in such a model may be regarded as the gauge field with respect to a gauged Z_2 subsymmetry of the group CPT. However, it should be mentioned that the assignments (1) not only assigns the fields to the elements of Z_2 but also implies that we arrange all fields into certain matrices. In fact, this arrangement is nothing to do with discrete gauge symmetry but for convenience in the forthcoming calculation. Of course, we must keep in mind that this arrangement is a working hypothesis and sometimes one should avoid some extra constraints coming from working hypotheses.

From the general framework we have developed in [1], it follows the generalized connection one-form

$$A(x,h) = A_\mu(x,h)dx^\mu + \frac{\lambda}{\mu}\Phi(x,h)\chi \qquad h \in Z_2 \tag{2}$$

where χ denotes χ^τ, and the generalized curvature two-form

$$\begin{aligned} F(h) &= dA(h) + A(h) \otimes A(h) \\ &= \frac{1}{2}F_{\mu\nu}(h)dx^\mu \wedge dx^\nu + \frac{\lambda}{\mu}F_{\mu\tau}(h)dx^\mu \otimes \chi + \frac{\lambda^2}{\mu^2}F_{\tau\tau}(h)\chi \otimes \chi. \end{aligned} \tag{3}$$

Using the above assignments, we get

$$\begin{aligned} F(x,e) &= F(x,r) \\ &= \frac{1}{2}\begin{pmatrix} L_{\mu\nu} & 0 \\ 0 & R_{\mu\nu} \end{pmatrix} dx^\mu \wedge dx^\nu + \frac{\lambda}{\mu}\begin{pmatrix} 0 & -D_\mu\phi \\ -D_\mu\phi^\dagger & 0 \end{pmatrix} dx^\mu \otimes \chi \\ &\quad + \frac{\lambda^2}{\mu^2}\begin{pmatrix} \phi\phi^\dagger - (\mu^2/\lambda^2) & 0 \\ 0 & \phi^\dagger\phi - (\mu^2/\lambda^2) \end{pmatrix} \chi \otimes \chi \end{aligned} \tag{4}$$

where

$$D_\mu\phi = \partial_\mu\phi + L_\mu\phi - \phi R_\mu. \tag{5}$$

Having these building blocks, we may get the generalized gauge invariant Lagrangian including both the bosonic part and the fermionic one as well as their interactions via generalized minimum coupling principle in the conventional way. From the field content (1), it follows Lagrangian of the ordinary type in gauge invariant models. On the other hand, as mentioned before, we may also introduce the generalized gauge invariant Lagrangian with respect to each element of Z_2 first, then take the Haar integral of them over Z_2.

For the Lagrangian of the bosonic sector with respect to each element of Z_2, we have

$$\begin{aligned}\mathcal{L}_{YM-H}(x,e) &= \mathcal{L}_{YM-H}(x,r)\\ &= -\frac{1}{4N_L}\mathrm{Tr}_L(L_{\mu\nu}L^{\mu\nu}) - \frac{1}{4N_R}\mathrm{Tr}_R(R_{\mu\nu}R^{\mu\nu}) + 2\eta\frac{\lambda^2}{\mu^2}\mathrm{Tr}(D_\mu\phi(x))(D^\mu\phi(x))^\dagger\\ &\quad - 2\eta^2\frac{\lambda^4}{\mu^4}\mathrm{Tr}\left(\phi(x)\phi(x)^\dagger - \frac{\mu^2}{\lambda^2}\right)^2 + \text{constant}\end{aligned} \tag{6}$$

where N_L and N_R are normalization constants, η is a metric parameter defined by $\eta = \langle\chi,\chi\rangle, \mathrm{Dim}(\eta) = \mu^2$. Here we suppose that both G_L and G_R are semi-simple.

For the fermionic sector, the Lagrangian with respect to each element of Z_2 may also be given as follows:

$$\begin{aligned}\mathcal{L}_F(x,e) &= \mathcal{L}_F(x,r)\\ &= i\bar{L}\gamma^\mu(\partial_\mu + L_\mu)L + i\bar{R}\gamma^\mu(\partial_\mu + R_\mu)R - \lambda(\bar{L}\phi R + \bar{R}\phi^\dagger L).\end{aligned} \tag{7}$$

It is easy to get the entire Lagrangian for the model:

$$\mathcal{L}(x) = \mathcal{L}_F(x,e) + \mathcal{L}_{YM-H}(x,e). \tag{8}$$

Obviously, there may still exist some constraints among the coupling constants and mass parameters, which will be explained for the concrete model.

We are now ready to reformulate the Weinberg–Salam model. For simplicity, we deal with the case of only one family of leptons. It is straightforward to generalize to the case of three families. For the case at hand, we have

$$\begin{aligned}&L(x) = \begin{pmatrix}\nu_l\\ l\end{pmatrix} \qquad R(x) = l_R \qquad \phi(x) = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}\\ &L_\mu = -ig\frac{\tau_i}{2}W^i_\mu + i\frac{g'}{2}B_\mu \qquad R_\mu = ig'B_\mu.\end{aligned} \tag{9}$$

Thus

$$\begin{aligned}L_{\mu\nu} &= -ig\frac{\tau_i}{2}W^i_{\mu\nu} + i\frac{g'}{2}B_{\mu\nu}\\ R_{\mu\nu} &= ig'B_{\mu\nu}\\ D_\mu\phi &= \left(\partial_\mu - ig\frac{\tau_i}{2}W^i_\mu - i\frac{g'}{2}B_\mu\right)\phi.\end{aligned} \tag{10}$$

Substituting these field contents into (6) and taking the Haar integral, we get the Lagrangian for the gauge bosons and Higgs in the Weinberg–Salam model:

$$\begin{aligned}\mathcal{L}_{YM-H}(x) = &-\frac{1}{4N_L}\frac{g^2}{2}W^i_{\mu\nu}W^{i\mu\nu} - \frac{1}{4N_Y}\frac{3g'^2}{2}B_{\mu\nu}B^{\mu\nu} + 2\eta\frac{\lambda^2}{\mu^2}\mathrm{Tr}(D_\mu\phi(x))(D^\mu\phi(x))^\dagger\\ &- 2\eta^2\frac{\lambda^4}{\mu^4}\mathrm{Tr}\left(\phi(x)\phi(x)^\dagger - \frac{\mu^2}{\lambda^2}\right)^2 + \text{constant}\end{aligned} \tag{11}$$

where $N_L(N_Y)$ is the normalization constant with respect to $SU(2)_L(U_Y)$. Since G_L is not semi-simple, here we should take this normalization instead of the one in (6). The normalization of the coefficients of each term results in

$$N_L = \frac{g^2}{2} \qquad N_Y = \frac{3g'^2}{2} \qquad \eta = \frac{\mu^2}{2\lambda^2}. \tag{12}$$

Similarly, we may get the Lagrangian for leptons $\mathcal{L}_F(x)$ as follows:

$$\begin{aligned}\mathcal{L}_F(x) = i\bar{L}(x)\gamma^\mu \left(\partial_\mu - ig\frac{\tau_i}{2}W^i_\mu + i\frac{g'}{2}B_\mu\right) L + i\bar{R}(x)\gamma^\mu(\partial_\mu + ig'B_\mu)R \\ - \lambda(\bar{L}(x)\phi(x)R(x) + \bar{R}(x)\phi(x)^\dagger L(x)).\end{aligned} \tag{13}$$

Thus, we obtain the entire Lagrangian for the Weinberg–Salam model as follows

$$\mathcal{L}(x) = \mathcal{L}_F(x) + \mathcal{L}_{YM-H}(x). \tag{14}$$

When $\mathrm{Tr}(\phi\phi^\dagger) = (\frac{\mu}{\lambda})^2$, the Higgs potential takes its minimum value and the continuous gauge symmetry will spontaneously be broken down. We now take the vacuum expectation value of ϕ as

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \qquad v = \sqrt{2}\frac{\mu}{\lambda} \tag{15}$$

and introduce a new field $\eta(x)$

$$\phi = \begin{pmatrix} 0 \\ [v + \eta(x)]/\sqrt{2} \end{pmatrix} \tag{16}$$

as well as the photon and Z boson via the Weinberg angle

$$\begin{aligned} A_\mu &= B_\mu \cos\theta_W + W^3_\mu \sin\theta_W \\ Z_\mu &= B_\mu \sin\theta_W - W^3_\mu \cos\theta_W \\ g\sin\theta_W &= g'\cos\theta_W = \frac{gg'}{\sqrt{g^2 + g'^2}} = e. \end{aligned} \tag{17}$$

Using (12), we get

$$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2} \left\{= \frac{N_Y}{3N_L + N_Y}\right\}. \tag{18}$$

And we have

$$\begin{aligned} &\mathrm{Tr}\left\{D_\mu\phi(D_\mu\phi)^\dagger - \frac{1}{2}\left(\phi\phi^\dagger - \frac{\mu^2}{\lambda^2}\right)^2\right\} \\ &\quad = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{g^2}{4}(v+\eta)^2 W^-_\mu W^+_\mu + \frac{1}{8}(g^2 + g'^2)(v+\eta)^2 Z_\mu Z_\mu \\ &\qquad - \frac{1}{2}\eta^2\left(v^2 + v\eta + \frac{\eta^2}{4}\right) + \text{constant}. \end{aligned} \tag{19}$$

It is easy to see that only A_μ and ν_l remain massless while fermion l together with $W^\pm$ and Z become massive and the following mass relations hold at the tree level:

$$
\begin{aligned}
M_{\text{fermion}} &= \mu \\
M_W &= \tfrac{1}{2} g v \\
M_Z &= \tfrac{1}{2}\sqrt{g^2 + g'^2}\, v = M_W / \cos\theta_w \\
M_{\text{Higgs}} &= v .
\end{aligned}
\tag{20}
$$

It is easy to see that all these relations at the tree level are the same as the ones for the Weinberg–Salam model except the last one for the Higgs mass. In order to avoid some extra constraints from the matrix arrangement (1), we have introduced two independent normalization constants N_L and N_Y. In fact, if we take $N_L = N_Y$ we could get a constraint for the Weinberg angle. In other words, the constraints in [1] are not essential but completely dependent on the working hypothesis. As for the Higgs mass given here at the tree level, it may also be released, say, by introducing a relative metric parameter in the metric on $M^4 \times Z^2$. Of course, it is worth it to see whether the fixed relative metric parameter in the metric on $M^4 \times Z^2$ survives the quantum correlations unlike other constraints [8].

Obviously, what we have done for the Weinberg–Salam model of the leptons are quite different from the non-commutative geometry approach to the model building [3–7] and can be easily generalized to the standard model for the electroweak–strong interaction. We will discuss these differences and deal with the standard model in [9].

One of the important points in this letter is the link between the Z_2-gauge symmetry in CPT and the Weinberg–Salam model as well as the standard model. It not only resolves the asymmetric field assignment with respect to the elements of Z_2-gauge symmetry in [1] if we apply it to the Weinberg–Salam model as well as the standard model, but also implies that the CPT symmetry as a whole probably should be gauged. Why the CPT symmetry should be gauged is in fact a simple but profound question similar to the questions why the Yang–Mills gauge fields should be introduced and why the Lorentz group should be gauged. As is well known, the content and implication of the CPT symmetry is very rich. Therefore, to gauge the entire CPT symmetry may shed light on some fundamental problems.

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